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RUNNING HEAD: Divergence point analysis

On using distributional analysis techniques for determining the onset of the influence of experimental variables

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Abstract

Much of the investigation of eye-movement control in visual cognition has focused on the influence of experimental variables on mean fixation durations. In the present paper we explored the convergence between two distributional analysis techniques that were recently introduced in this domain. First, Staub, White, Drieghe, Hollway and Rayner, (2010) proposed fitting the ex-Gaussian distribution to individual participants' data in order to ascertain whether a variable has a rapid or a slow influence on fixation durations. Second, the Divergence Point Analysis (DPA) procedure was introduced by Reingold, Reichle, Glaholt and Sheridan (2012, Reingold & Sheridan, 2014) in order to determine more precisely the earliest discernible impact of a variable on the distribution of fixation durations by contrasting survival curves across two experimental conditions and determining the point at which the two curves begin to diverge. In the present paper we introduced a new version of the DPA procedure which is based on ex-Gaussian fitting. We evaluated this procedure by re-analysing data obtained in previous empirical investigations as well as by conducting a simulation study. We demonstrated that the new ex-Gaussian DPA technique produced estimates that were consistent with estimates produced by prior versions of DPA procedure, and in the present simulation, the ex-Gaussian DPA procedure produced somewhat more accurate individual participant divergence point estimates. Based on the present findings we also suggest guidelines for best practices in the use of DPA techniques.

The empirical investigation of the time-course of mental processes often involves the analysis of the latency with which a variable impacts performance on perceptual and cognitive tasks. In particular, extensive research manipulated variables that influence fixation duration and reaction time (RT) data. While much of this research was focused on the analysis of measures of central tendency such as the mean or median of response latency, there is growing realization that distributional analyses are inherently more suitable for determining the time-course of the influence of experimental manipulations. In the present paper, we examine and illustrate this trend in the context of the study of eye-movement control in reading. Specifically, we explore two distributional analysis techniques that were recently introduced in this domain. First, Staub, White, Drieghe, Hollway and Rayner, (2010) proposed fitting the ex-Gaussian distribution to individual participants' data in order to ascertain whether a variable has a rapid or a slow influence on fixation duration. Second, the Divergence Point Analysis (DPA) procedure was introduced by Reingold, Reichle, Glaholt and Sheridan (2012, Reingold & Sheridan, 2014) in order to determine more precisely the earliest discernible impact of a variable on the distribution of fixation duration by contrasting survival curves across two experimental conditions and determining the point at which the two curves begin to diverge (i.e., the divergence point).

The main goal of the present paper was to explore the convergence between the ex-Gaussian fitting method and the DPA procedure. To accomplish this goal, we devised a new version of the DPA procedure in which divergence point estimates are derived by contrasting survival curves that were produced using the best-fitting ex-Gaussian parameters rather than using the nonparametric bootstrapping method which was employed in previous implementations of the DPA procedure. A second goal of the present paper was to evaluate the strengths and weaknesses of several versions of the DPA procedure and suggest guidelines for best practices in

the use of DPA techniques. Accordingly, we begin by briefly describing the controversy which motivated the introduction of these distributional analysis methods and the evidence which was obtained in studies implementing these procedures. Using a re-analysis of data from prior studies as well as a simulation study, we then compare the new ex-Gaussian DPA procedure with prior versions of the DPA technique.

Over the past 4 decades, the investigation of eye-movement control in reading generated a substantial body of findings as well as considerable controversy (see Rayner, 1998, 2009 for reviews). Although, it is now generally accepted that mean fixation duration is influenced by lexical and/or linguistic variables such as word frequency (Inhoff & Rayner, 1986; Rayner & Duffy, 1986), predictability (Ehrlich & Rayner, 1981), lexical ambiguity (e.g., Duffy, Morris, & Rayner, 1988; Rayner & Duffy, 1986) and age of acquisition (e.g., Juhasz & Rayner, 2006), there is an ongoing controversy concerning the time course of lexical influences. Specifically, contemporary models advocating primarily visual/oculomotor control of fixation durations in reading assume that lexical effects are limited to a small subset of long fixations and that the vast majority of reading fixations are unaffected by these variables. In marked contrast, other models argue in favor of the validity of the *direct cognitive-control hypothesis*, which suggests that the lexical properties of the fixated word produce a rapid influence on fixation duration impacting a substantial proportion of reading fixations (for a detailed review of this controversy see Reingold, Sheridan, & Reichle, 2015).

Staub et al. (2010) argued that ex-Gaussian fitting might provide a useful method for empirically testing the validity of conflicting assumptions concerning the time-course of the influence of lexical variables on fixation duration. The typical unimodal (i.e., single peaked) distribution of fixation duration in reading resembles the Gaussian/normal distribution yet

presents a clear rightward (positive) skew. That is, there are more fixations near the center of the distribution than on either tail, and the right tail of the distribution is more densely populated than the left tail. Consequently, the ex-Gaussian distribution, which was previously established to provide good fits for RT distributions (see Balota & Yap, 2011 for a review), might be suitable for modeling the empirical distributions of fixation durations. The ex-Gaussian distribution is a convolution (sum) of two stochastically independent random variables: a normally distributed random variable and an exponentially distributed random variable. The ex-Gaussian distribution can be fully specified with three parameters: μ (Mu, the mean of the Gaussian component), σ (Sigma, the standard deviation of the Gaussian component), and τ (Tau, the mean of the Exponential component). Importantly, the mean of the ex-Gaussian distribution equals $\mu + \tau$, and its variance equals $\mu^2 + \tau^2$.

To illustrate the contributions of the Gaussian and Exponential components to the overall shape of the ex-Gaussian distribution, Figure 1 shows the probability density functions (Panels a, c and e) of 3 ex-Gaussian distributions and their Gaussian and Exponential components. The comparison between Panel a and Panel c illustrates the impact of changing the mu parameter while holding the sigma and tau parameters constant. Such a mu effect results in a shift of the ex-Gaussian distribution without altering its shape. In contrast, the degree of skew of the ex-Gaussian distribution is altered when the tau parameter is changed while the mu and sigma parameters are held constant (Panel e vs. Panel c of Figure 1). To better illustrate the impact of a tau manipulation, Figure 1 also includes the corresponding survival curves (Panels b, d and f). Survival curves provide an alternative way of plotting the ex-Gaussian distribution of fixation durations. Specifically, at time *t*, the percentage of fixations with a duration greater than *t* is referred to as the percent *survival* at time *t*. When *t* equals zero, survival is at one hundred

percent, but then monotonically declines as *t* increases. As clearly illustrated in Figure 1 (see cross-hatched areas in Panels b, d and f), the contribution of the Exponential component (i.e., the tau effect) can be seen as soon as the survival rate of the Gaussian component begins to decline. Thus, although the impact of a manipulation of the tau parameter is most visible at the right tail of the density function, the onset of such an effect occurs much earlier.

Based on the above properties of the ex-Gaussian distribution, Staub et al. (2010) argued that a distributional shift effect (i.e., a difference in mu between conditions) indicates that a variable has an early acting influence on the majority of fixation durations. Using this logic, Staub et al. (2010) fitted an ex-Gaussian distribution to individual participants' fixation duration distributions on both high- and low-frequency target words. These fits demonstrated that the low-frequency distribution was significantly shifted to the right of the high-frequency distribution indicating that this lexical variable has an impact on both short and long fixations as predicted by the direct cognitive-control hypothesis. Convergent evidence for this conclusion was derived from studies employing the DPA procedure which was introduced by Reingold et al. (2012). This distributional analysis technique was aimed at determining the earliest discernible impact of a variable by contrasting survival curves across two experimental conditions and using a bootstrap resampling procedure (Efron & Tibshirani, 1994) for estimating the point at which the two curves begin to diverge. As reviewed by Reingold et al. (2015), lexical variables including word frequency, predictability and lexical ambiguity were demonstrated to produce rapid influences on fixation durations, as indicated by a significant mu effect (i.e., a shift effect) for all of these variables, and by DPA analysis results showing that the first discernible influence of these variables occurs approximately 110-150 ms after fixation onset.

It is important to note that these rapid divergence point estimates for lexical variables were demonstrated despite the fact that the version of the DPA procedure introduced by Reingold et al. (2012) (henceforth, the Original DPA procedure) incorporated very conservative criteria for estimating divergence points in order to protect against making a Type I error (i.e., erroneously detecting a divergence point). However, as demonstrated by Reingold and Sheridan (2014), while the Original DPA procedure produces fairly accurate estimates of divergence points in studies using a large number of observations per condition, with lower experimental power this version of the procedure produces divergence point estimates which occur later than the actual point of divergence. In part to correct for that bias, Reingold and Sheridan (2014) introduced 2 modified DPA procedures: 1) the Individual Participant DPA procedure (IP-DPA) that was designed for computing divergence point estimates for each participant in the sample, and 2) the Confidence Interval DPA procedure (CI-DPA) which like the Original DPA procedure provides a divergence point estimate based on group data but also permits the computation of confidence intervals for divergence point estimates. Reingold and Sheridan (2014) reported that the modified DPA procedures performed much better than the Original DPA procedure under conditions of low experimental power. In addition, with a sufficient number of items per condition, the IP-DPA procedure showed promise as an accurate method for measuring individual differences in divergence points.

In addition to the 3 previously employed versions of the DPA procedure (Original, IP-DPA, CI-DPA), in the present paper we estimated divergence points based on the best fitting ex-Gaussian parameters for each participant and condition (ex-Gaussian DPA). Specifically, the ex-Gaussian parameters could be used to derive continuous probability density functions and survival curves. Consequently, bootstrapping is not required for the determination of the

divergence point between 2 survival curves. Instead we defined the divergence point as the smallest value of fixation duration for which survival percent in the slow condition was at least 1.5% greater than the survival percent in the fast condition. As suggested by Reingold and Sheridan (2014), given the typical experimental power employed in the literature, this 1.5% criterion permits reliable detection of divergence due to non-trivial experimental effects while minimizing the risk of falsely detecting divergence under noisy low power conditions.

To begin exploring the ex-Gaussian DPA procedure, we re-analyzed the distributions of first-fixation durations from several studies examining the influence of 3 lexical variables: word frequency (Reingold et al. 2012, valid preview condition), predictability (Sheridan & Reingold, 2012a), and lexical ambiguity (Sheridan & Reingold, 2012b). Figure 2 (Panels a, c, and e) illustrates a range of progressively longer divergence point estimates that were obtained using the best fitting ex-Gaussian parameters for 3 different participants. More importantly, as shown in Table 1, divergence point estimates that were derived using the ex-Gaussian DPA procedure were consistent with estimates generated using previously employed versions of the DPA procedure (Original, IP-DPA, CI-DPA). Furthermore, as shown in Figure 2 (Panels b, d and f), for each lexical variable the estimates derived from the ex-Gaussian DPA and the IP-DPA procedures were highly correlated across participants. Next, we conducted a Monte Carlo simulation study in order to more extensively evaluate the reliability and accuracy of the ex-Gaussian DPA procedure and previously employed versions of the DPA procedure.

Simulation Study

The re-analysis of data from prior experiments investigating the influence of lexical variables on fixation duration (see Table 1, Figure 2) demonstrated substantial variability in divergence point estimates across participants. For the purpose of the simulation study we

selected 28 subjects from prior studies for which ex-Gaussian divergence point estimates varied between 55 ms and 190 ms (increasing across participants by 5 ms steps: 55 ms, 60 ms, 65 ms,....,190 ms; Mean = 122.5). The best fitting ex-Gaussian parameters for the slow and fast conditions of each of these 28 subjects are shown in supplemental materials. These 28 contrasts between 2 continuous ex-Gaussian distributions served as the populations which were used in the present simulation study. Specifically, 100 samples of 28 simulated participants with either 30, 60 or 120 observations (i.e., fixations) per condition (i.e., fast vs. slow) were created by randomly sampling fixation durations from these populations. For each of the 8,400 combinations of simulated participants (1-28) by number of fixations per condition (30, 60, 120) by sample number (1-100), we also created a baseline condition in which observations from the fast and slow conditions were pooled together and randomly shuffled (i.e., the shuffled condition was created by a random reassignment of observations to fast and slow conditions). For DPA procedures that are based on group data (Original-DPA, CI-DPA), the shuffled baseline condition was used to examine the probability of falsely detecting divergence where none exists. In addition, for DPA procedures that are based on individual participant data (IP-DPA, ex-Gaussian DPA) the shuffled condition was used to provide a baseline for the evaluating magnitude of the correlations between DPA estimates and divergence point values in the populations.

Method

For each of the 300 samples of 28 simulated participants we computed DPA estimates for both shuffled and unshuffled data using the Original-DPA and CI-DPA procedures. For each of the 8,400 participants across these samples we computed DPA estimates for both shuffled and

unshuffled data using the IP-DPA and ex-Gaussian-DPA procedures. The Estimates were computed as follows (see supplemental materials for MATLAB code):

Original-DPA procedure – Following Reingold et al. (2012) we used 10,000 iterations of random resampling of fixations for each participant and condition. For each iteration of the bootstrap procedure, individual participant's survival curves were then computed and averaged across participants. Next, the value for each 1-ms bin ranging from 1 to 600 ms in the fast condition was subtracted from the corresponding value in the slow condition. The obtained differences for each bin were then sorted in order of magnitude. The range between the 5th and the 9,995th value was then defined as the confidence interval of the difference for each bin. Time bins for which the lower bound of the confidence interval of the difference between the slow and fast survival curves was greater than zero were considered to represent a significant difference between curves. The divergence point was then defined as the earliest significant difference point that was part of a run of five consecutive significant difference points.

CI-DPA procedure – This procedure was identical to the method used by Reingold et al. (2012) procedure with the exception that the divergence point was calculated for each iteration rather than once across all iterations. Specifically, in the CI-DPA procedure, in each iteration, the divergence point estimate was defined as the first 1-ms bin in a run of five consecutive bins in which the survival percent in the slow condition was at least 1.5% greater than the survival percent in the fast condition. Across the 1,000 iterations, divergence point estimates were then sorted from the smallest to the largest value and the 25th and 975th values constituted the 95% confidence interval. In addition, the median of the 1,000 divergence point values was used as the divergence point estimate for the sample.

Individual Participant DPA (IP-DPA) procedure – Following Reingold and Sheridan (2014), we derived the individual divergence estimates by performing the following sequence of steps separately for each subject's fixation duration data. For each of 1,000 bootstrap iterations, regardless of the number of fixations that were available for a given subject (i.e., 30, 60 or 120), 1200 fixations were randomly sampled with replacement from the pool of fixations corresponding to the slow condition and from the pool of fixations corresponding to the fast condition. Both sets of 1200 fixations were sorted from the shortest to the longest duration value and then paired (i.e., $s_1 f_1, s_2 f_2, s_3 f_3 \ldots s_{1200} f_{1200}$). For both the slow and fast condition, the sorted order of fixations specified the sequence in which fixations were terminated, and in turn the "death" of a single fixation decreased the survival percent by the minimum possible decrement (i.e., *1/1200*100*). Consequently, the process of sorting by fixation duration created 1200 survival percent bins (for bin *i* survival percent equalled *100-i/1200*100)*. For each of the 1200 survival percent bins, the difference between the duration of the slow minus the fast fixation duration was computed (for bin *i*, this difference equalled $s_i - f_i$). Next, we identified the first survival percent bin in a run of 100 consecutive bins in which the value of this difference was positive ($s_i > f_i$). Finally, the average duration of the pair of fixations corresponding to that bin was defined as the divergence point value for each iteration. Iterations in which a divergence point value was not identified were discarded and the median value across the remaining iterations was then defined as the divergence point estimate for that individual. Participants for which a divergence point value was obtained in less than 50% of iterations were deemed unreliable and were excluded.

Ex-Gaussian DPA procedure – To compute the ex-Gaussian DPA divergence estimates, we performed the following steps separately for each subject's fixation duration data. First, we

fitted the ex-Gaussian distribution to the each subject's fixation duration data using an algorithm known as *quantile maximum likelihood estimation* (*QMPE*; Cousineau, Brown, & Heathcote, 2004; Heathcote, Brown, & Mewhort, 2002). The fixation duration data for each condition (i.e., fast and slow) were fitted separately, and all fits successfully converged. These ex-Gaussian parameters were then used to derive continuous probability density functions and survival curves for the fast and slow conditions. Using these survival curves, we defined the divergence point for each subject as the smallest value of fixation duration for which survival percent in the slow condition was at least 1.5% greater than the survival percent in the fast condition.

Results

We begin by summarizing the findings for DPA procedures that are based on group data (Original-DPA, CI-DPA), followed by an exploration of the results for DPA procedures that are based on individual participant data (IP-DPA, ex-Gaussian DPA).

Group based DPA estimates - Figure 3 (Top Panel) illustrates the average DPA estimates produced by all 4 versions of the DPA procedure as a function of sample size. For group based DPA estimates, the pattern of findings replicated the results reported by Reingold and Sheridan (2014). Specifically, while the Original-DPA procedure was strongly influenced by sample size ($F(2, 297) = 96.44$, $p < 0.001$), the CI-DPA estimates did not vary as a function of sample size ($F(2, 297) = 1.45$, $p > 0.23$). In particular, under low experimental power (i.e., a small number of observations), the Original-DPA procedure produced estimates of divergence points that were delayed relative to the estimates that were produced when the experimental power was more substantial. No such bias was seen for estimates produced by the CI-DPA procedure.

As discussed by Reingold and Sheridan (2014), this bias was built into the Original-DPA procedure in order to avoid erroneously detecting a divergence point. The shuffled condition in

the present simulation provided a direct test of how successfully group based DPA procedures handle the absence of divergence. Importantly, while the Original-DPA procedure produced a divergence point estimate for each sample with unshuffled data, this procedure failed to produce a DPA estimate in the vast majority of samples with shuffled data (96.7%). In contrast, the CI-DPA procedure did not offer a clear method for avoiding the false detection of a divergence where none existed. For this procedure, while shuffled data resulted in a failure to diverge in about 22% of iterations on average (with unshuffled data such a failure was very rare occurring in less than 1% of iterations on average), the majority of iterations produced a false divergence. Based on the results from the present simulation, we would recommend using both procedures simultaneously: The Original-DPA procedure to rule out false detection of divergence and the CI-DPA procedure to produce the actual divergence point estimates.

Another important aspect of group based DPA estimates is related to the present finding (See Figure 3, Top Panel) that mean CI-DPA estimates (~106 ms) were consistently earlier than the average population divergence point values (122.5 ms). As pointed out by Reingold and Sheridan (2014), DPA procedures that are based on group data detect a divergence at the point at which participants in the sample with early divergence point estimates begin to significantly influence the group survival curves. To illustrate this point we subdivided each of the 300 samples into two subsamples of 14 simulated participants: The Low-variance group (with population divergence point values between 90 ms to 155 ms; Mean = 122.5) and the High-variance group (with population divergence point values between 55 ms to 85 ms and between 160 ms to 190 ms; Mean = 122.5). Despite the fact that the mean population divergence point values were equated between these groups, if CI-DPA estimates are influenced by participants in the sample with early divergence point estimates then the High-variance group should produce

earlier divergence point estimates than the Low-variance group. As shown in Figure 3 (Bottom Panel), consistent with this prediction, regardless of sample size, the DPA estimates for the High-variance group were significantly earlier than the DPA estimates for the Low-variance group (all *t*s > 9.89, all *p*s < .001).

Individual Participant based DPA estimates - as shown in Figure 3 (Top Panel), for each sample we also computed the average of the divergence point estimates across the 28 simulated participants and compared it with the average population divergence point value (i.e., 122.5 ms). Generally, both the ex-Gaussian DPA and IP-DPA procedures produced estimates that were fairly accurate even with low experimental power (i.e., 30 fixations per condition). However, regardless of the number of fixations per condition the average estimate produced by both procedures was significantly larger than the population value (all *t*s > 126.40, all *p*s < .001). In addition, for 30 fixation per condition, and to a lesser extent for 60 fixations per condition, the ex-Gaussian DPA estimates were significantly more accurate than the IP-DPA estimates (both *t*s > 15.02, both *p*s < .001), while the pattern was reversed (albeit very slightly) for 120 fixations per condition ($t(99) = 2.69$, both $p < .01$).

In order to evaluate the accuracy with which the variability in divergence point estimates reflect actual individual differences, for each sample of 28 simulated participants we computed the correlations between the divergence point values in the population and divergence point estimates produced by either the ex-Gaussian DPA procedure or the IP-DPA procedure. In addition, we also computed the correlations between ex-Gaussian DPA estimates and the corresponding IP-DPA estimates. All of these correlations were computed for both unshuffled data as well as for the shuffled data baseline condition. Note that for computing the mean correlation across 100 samples, we first converted Pearson's r's to the normally distributed

Fisher's z and then the mean Fisher's z value was converted back to Pearson r. In addition, Fisher's z values rather Pearson's r's were used in all statistical tests comparing differences in the magnitude of correlations across conditions.

Figure 4, shows both the means (Panels b, d, and f) and the distributions (Panels a, c, and e) of these correlations as a function of the number of fixations per condition. As can be seen in the figure, regardless of the number of fixations per condition, the ex-Gaussian DPA estimates performed slightly but significantly better than the IP-DPA estimates as predictors of the population divergence point values ($F(2, 297) = 44.76$, $p < 0.001$). More importantly, both procedures performed well producing moderate correlations with 30 fixations per condition and substantially larger correlations with 60 fixations per condition (i.e., a level experimental power which more closely resembles previous empirical investigations; both ts > 6.20, both ps < .001). The magnitude of the correlation was further increased with 120 fixations per condition (both ts > 5.35, both ps < .001). Finally, the mean correlation between the ex-Gaussian DPA and the IP-DPA estimates was very high even for shuffled data (although the correlations were slightly larger for unshuffled data ($F(2, 297) = 51.06$, $p < 0.001$), and there was only a marginal effect of the number of fixations per condition ($F(2, 297) = 2.74$, $p = 0.07$).

Discussion

In the present paper we extended the DPA procedure which was introduced by Reingold et al. (2012) as a method for investigating the direct cognitive control hypothesis. Given the persistent skepticism concerning the feasibility of a direct cognitive control mechanism, the Original-DPA procedure incorporated a conservative criterion that made it more difficult to obtain evidence supporting this hypothesis. Despite this built in bias, the findings that emerged from this paradigm provided strong support for the validity of the direct cognitive control

hypothesis (for a review see Reingold, Sheridan, & Reichle, 2015). Reingold and Sheridan (2014) demonstrated the bias in the estimates derived from the Original-DPA procedure, and introduced the CI-DPA and IP-DPA versions of this procedure which were less negatively impacted by lower experimental power, and consequently provided more accurate DPA estimates. In the present simulation, we replicated the findings reported by Reingold and Sheridan (2014), but we also demonstrated that the Original-DPA procedure was very successful in not producing false divergence with the randomly shuffled baseline condition. Consequently, we strongly recommend that the Original-DPA procedure be used as a validity check prior to utilizing the other versions of the procedure. More specifically, if the Original-DPA procedure fails to produce a divergence point then the DPA estimates produced using the other versions of the procedure would be deemed invalid. By using multiple procedures, it is possible to obtain more accurate DPA estimates while at the same time protecting against the erroneous detection of divergence where none exists.

Another contribution of the present study is related to the introduction of the ex-Gaussian DPA procedure. In particular, the findings from the simulation study and the results from the re-analysis of empirical data provided strong support for the potential usefulness of this version of the DPA procedure. The ex-Gaussian DPA estimates were consistent with estimates produced by prior versions of the DPA procedure, and under the conditions used in the present simulation, the ex-Gaussian DPA procedure produced individual participant estimates which were somewhat more accurate than those produced by IP-DPA procedure. However, it is important to recognize that the fact that ex-Gaussian populations were used in the present simulation likely contributed to the accuracy of the estimates derived from ex-Gaussian DPA procedure. Future simulations could explore the performance of the ex-Gaussian DPA procedure under conditions in which the

populations deviate from the ex-Gaussian model. In this regard, the nonparametric bootstrapping approach that is incorporated into prior versions of the DPA procedure has an advantage because it does not rely on distributional assumptions and/or on the availability of analytic formulas. Consequently, to the extent that the sample is representative of the population, the bootstrap resampling technique offers a very powerful and flexible tool for making statistical inferences. Thus, in order to obtain accurate and reliable estimates of divergence point for each individual participant, we would strongly recommend that researchers use both the ex-Gaussian DPA and the IP-DPA procedures as well as maximize the number of observations per condition (i.e., use adequate experimental power). In addition, the correlation between the DPA estimates produced by these 2 methods would potentially provide an excellent measure of reliability.

More generally, it is also interesting to consider the use of ex-Gaussian fitting to derive divergence point estimates in the context of the argument advocated by Staub et al. (2010) that an experimental manipulation which impacts the mu parameter necessarily implies that the variable manipulated has a rapid influence on fixation duration. Employing the ex-Gaussian DPA procedure might help clarify some of the considerations which are relevant for interpreting differences in ex-Gaussian parameters across conditions. To illustrate this, in Figure 5, ex-Gaussian survival curves are presented for contrasts between a fast experimental condition with mean fixation duration of 203 ms (mu=157.87, sigma=39.98 and tau=45.26) versus 3 different slow conditions with mean fixation duration of 223 ms (i.e., all contrasts represent a 20 ms experimental effect). In the first contrast (Panel a) the slow condition was created by adding 20 ms to the mu parameter of the fast condition while holding the tau and sigma constant. As discussed above such a mu effect manifests as a shift in the survival curves. In addition, consistent with the argument of Staub et al. (2010) this mu effect also produced a rapid

divergence point estimate (100 ms). In the second contrast (Panel b) the slow condition was created by adding 20 ms to the tau parameter of the fast condition while holding mu and sigma constant. This tau effect produced the expected increase in skew, coupled with a later divergence point estimate (123 ms). At first blush, the rather small difference in divergence point estimates between a mu effect and a tau effect might appear to contradict the common conceptualization of a tau effect as a "late" effect. However, recall that although the impact of a manipulation of tau is most visible for longer fixation duration values, the onset of a tau effect occurs much earlier (see Figure 1 for an illustration).

Furthermore, it is important to note that unlike the mu effect in the first contrast, the tau effect in the second contrast also increased the variance of the ex-Gaussian distribution of the slow condition (first contrast: *SD*=60.39; second contrast: *SD*=76.53). In the third contrast (Panel c) the slow condition was created by adding 20 ms to the mu parameter of the fast condition as well as by increasing the sigma parameter from 40 to 61.71. This mu plus sigma effect produced a slow condition which had the same mean and standard deviation as the slow condition produced by the tau effect (Panel b). Importantly, despite the difference in mu between the fast and slow conditions, the third contrast resulted in a longer divergence point estimate (152 ms) than the estimates obtained for the other 2 contrasts. Therefore, it is clear that unlike the argument of Staub et al. (2010), a mu effect is neither a necessary nor a sufficient condition for producing early divergence.

The above illustration highlights the importance of distinguishing between 2 different approaches for motivating the usefulness of ex-Gaussian fitting of response latency data. Historically, the ex-Gaussian distribution was proposed as a theoretical approach for modelling the RT distribution (e.g., Hohle, 1965; for reviews see Heathcote, Popiel, & Mewhort, 1991;

Matzke & Wagenmakers, 2009, Schwarz, 2001). According to such an approach the Gaussian and Exponential components of the ex-Gaussian distribution were assumed to reflect the operation of different processes or stages which operate sequentially to produce the observed latency data. More recently, such direct mapping between ex-Gaussian parameters and underlying processes has been widely criticized (e.g., Heathcote et al., 1991; Matzke & Wagenmakers, 2009, Sternberg, 2014). As Heathcote et al. (1991) argued, "Although the ex-Gaussian model describes RT data successfully, it does so without the benefit of an underlying theory" (p. 346). Instead the ex-Gaussian distribution has been commonly employed as merely a descriptive tool used to provide a 3-parameter summary of RT distributions in order to test the predictions of competing theoretical models by going beyond measures of central tendency.

In light of the above discussion, we strongly argue that DPA procedures and ex-Gaussian fitting could provide an important tool for converging on a methodology for investigating eye-movement control in visual cognition and for providing empirical benchmarks for the development and testing of related computational models (for such an application see Sheridan & Reichle, 2015; Sheridan, Reichle, & Reingold, 2016). However, we would like to emphasize that our approach to DPA procedures (including the ex-Gaussian DPA) technique is purely descriptive. That is, we do not *a priori* assume that prior to the divergence point, processing is identical across the 2 experimental conditions that are being contrasted. In addition, we do not assume that following the divergence point the difference between conditions is solely due to the operation of a particular process or stage of processing. Instead, we aim to determine as precisely as possible the point at which a particular experimental manipulation produces a discernable impact on the distribution of fixation durations. The controversy concerning the time course of lexical influences on fixation durations provides an excellent illustration for the need

for such a divergence point estimate and we believe that we demonstrated that the DPA approach shows promise and merits further evaluation and development.

Supplemental Material

The Supplemental Material can be found at the address (online address to be filled in)

References

Balota, D. A., & Yap, M. J. (2011). Moving Beyond the Mean in Studies of Mental Chronometry: The Power of Response Time Distributional Analyses. *Current Directions in Psychological Science*, *20*(3), 160–166. http://doi.org/10.1177/0963721411408885

Cousineau, D., Brown, S., & Heathcote, A. (2004). Fitting distributions using maximum likelihood: methods and packages. *Behavior Research Methods, Instruments, & Computers*, *36*(4), 742–756. Retrieved from http://www.ncbi.nlm.nih.gov/pubmed/15641420

Duffy, S. A., Morris, R. K., & Rayner, K. (1988). Lexical ambiguity and fixation times in reading. *Journal of Memory and Language*, *27*(4), 429–446. http://doi.org/10.1016/0749-596X(88)90066-6

Efron, B., & Tibshirani, R. J. (1994). *An introduction to the bootstrap*. Boca Raton, FL: Chapman & Hall.

Ehrlich, S. F., & Rayner, K. (1981). Contextual effects on word perception and eye movements during reading. *Journal of Verbal Learning and Verbal Behavior*, *20*(6), 641–655. Retrieved from http://www.sciencedirect.com/science/article/pii/S0022537181902206

Heathcote, A., Brown, S., & Mewhort, D. J. K. (2002). Quantile maximum likelihood estimation of response time distributions. *Psychonomic Bulletin & Review*, *9*(2), 394–401. Retrieved from http://www.springerlink.com/index/p7241345l444446n.pdf

Heathcote, A., Popiel, S. J., & Mewhort, D. J. (1991). Analysis of response time distributions: An example using the Stroop task. *Psychological Bulletin*, *109*, 340–347.

Hohle, R. H. (1965) Inferred components of reaction times as functions of foreperiod duration. *Journal of Experimental Psychology*, *69*, 382-386.

Inhoff, A. W., & Rayner, K. (1986). Parafoveal word processing during eye fixations in reading: Effects of word frequency. *Perception & Psychophysics*, *40*(6), 431–439.

Juhasz, B. J., & Rayner, K. (2006). The role of age of acquisition and word frequency in reading: Evidence from eye fixation durations. *Visual Cognition*, *13*, 846–863. Retrieved from http://www.tandfonline.com/doi/abs/10.1080/13506280544000075

Matzke, D. & Wagenmakers, E.-J. (2009) *Psychonomic Bulletin & Review*, *16*, 798-817.

Rayner, K. (1998). Eye movements in reading and information processing: 20 years of research. *Psychological Bulletin*, *124*(3), 372–422. Retrieved from http://psycnet.apa.org/psycinfo/1998-11174-004

Rayner, K. (2009). Eye movements and attention in reading, scene perception, and visual search. *Quarterly Journal of Experimental Psychology*, *62*(8), 1457–1506. http://doi.org/10.1080/17470210902816461

Rayner, K., & Duffy, S. A. (1986). Lexical complexity and fixation times in reading: Effects of word frequency, verb complexity, and lexical ambiguity. *Memory & Cognition*, *14*(3), 191–201. Retrieved from http://www.ncbi.nlm.nih.gov/pubmed/3736392

Reingold, E. M., Reichle, E. D., Glaholt, M. G., & Sheridan, H. (2012). Direct lexical control of eye movements in reading: Evidence from a survival analysis of fixation durations. *Cognitive Psychology*, *65*, 177–206. http://doi.org/10.1016/j.cogpsych.2012.03.00

Reingold, E. M., & Sheridan, H. (2014). Estimating the divergence point: a novel distributional analysis procedure for determining the onset of the influence of experimental variables. *Frontiers in Psychology*, *5*(December), 1–10. http://doi.org/10.3389/fpsyg.2014.01432

Reingold, E. M., Sheridan, H., & Reichle, E. D. (2015). Direct lexical and nonlexical control of fixation duration in reading. In A. Pollatsek, & R. Treiman (Eds.), *The Oxford Handbook of Reading* (pp. 261–276). Oxford University Press, Oxford, UK.

Schwarz, W. (2001) The ex-Wald distribution as a descriptive model of response times. *Behavior Research Methods, Instruments, & Computers*, *33*, 457-469.

Sheridan, H., & Reichle, E. D. (2015). An analysis of the time course of lexical processing during reading. *Cognitive Science*. doi:10.1111/cogs.12246

Sheridan, H., & Reingold, E. M. (2012a). The time course of predictability effects in reading: Evidence from a survival analysis of fixation durations. *Visual Cognition*, *20*, 733–745. http://doi.org/10.1080/13506285.2012.693548

Sheridan, H., & Reingold, E. M. (2012b). The time course of contextual influences during lexical ambiguity resolution: Evidence from distributional analyses of fixation durations. *Memory & Cognition*, *40*, 1122–31. http://doi.org/10.3758/s13421-012-0216-2

Sheridan, H., Reingold, E. M. & Reichle, E. D. (2016). Why does removing inter-word spaces produce reading deficits? The role of parafoveal processing. *Psychonomic Bulletin & Review.* Advance online publication, doi: 10.3758/s13423-015-0997-y.

Staub, A., White, S. J., Drieghe, D., Hollway, E. C., & Rayner, K. (2010). Distributional effects of word frequency on eye fixation durations. *Journal of Experimental Psychology: Human Perception and Performance*, *36*(5), 1280–93. http://doi.org/10.1037/a0016896

Sternberg, S. (2014). Reaction times and the ex-Gaussian distribution: When is it appropriate? Retrieved from http://www.psych.upenn.edu/˜saul/exgauss.stages.pdf

Figure captions

Figure 1. An illustration of probability density functions (Panels a, c and e) and survival curves (Panels b, d and f) of the Gaussian and Exponential components of 3 ex-Gaussian distributions. A comparison of the middle row and the top row illustrates the impact of changing the mean of the Gaussian component (mu), and a comparison of the middle row and the bottom row illustrates the impact of changing the mean of the Exponential component (tau). The cross-hatched areas illustrate the impact of the Exponential component on the survival curve of the ex-Gaussian distribution (see text for details).

Figure 2. An illustration of the ex-Gaussian DPA procedure (Panels a, c and e depict progressively longer divergence point estimates as shown by the vertical line in each panel), and scatterplots illustrating the correlations between divergence point estimates produced by the ex-Gaussian DPA and the IP-DPA procedures for the re-analysis of data from studies examining the influence of 3 lexical variables (Panel b: word frequency - Reingold et al. 2012, valid preview; Panel d: predictability - Sheridan & Reingold, 2012a, and Panel f: lexical ambiguity - Sheridan & Reingold, 2012b) (see text for details).

Figure 3. The Top Panel displays the mean (across 100 random samples) of the divergence point estimates (averaged across the simulated participants in each sample) by DPA procedure, as a function of the sample size. The dashed horizontal line in the Top Panel indicates the average divergence point value in the population (122.5 ms). The Bottom Panel displays the CI-DPA estimates for the High-Variance group versus the Low-Variance group, as a function of the sample size. The horizontal line inside each box plot indicates the mean value, and the upper and lower bounds represent the maximum and minimum values respectively. See text for details.

Figure 4. The distributions of the correlations between the divergence point values in the populations and divergence point estimates produced by either the ex-Gaussian DPA procedure or the IP-DPA procedure, as a function of the sample size (left column, Panels a, c, and e) and average correlation coefficients for the shuffled versus unshuffled conditions (right column, Panels b, d, and f) as a function of the sample size (see text for details).

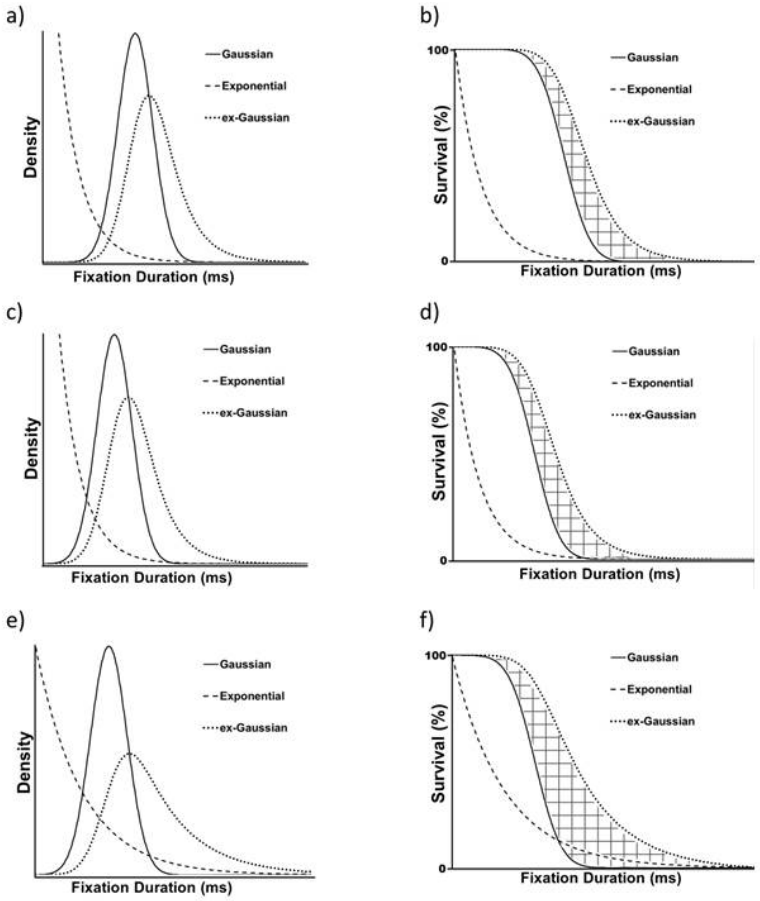
Figure 5. The survival curves for contrasts between a fast experimental condition with mean fixation duration of 203 ms (mu=157.87, sigma=39.98 and tau=45.26) versus 3 different slow conditions with mean fixation duration of 223 ms. These 3 contrasts represent: 1) a mu effect (Panel a; slow condition: mu=177.87, sigma=39.98 and tau=45.26), 2) a tau effect (Panel b; slow condition: mu=157.87, sigma=39.98 and tau=65.26), and 3) a mu plus sigma effect (Panel c; slow condition: mu=177.87, sigma=61.71 and tau=45.26) (see text for details).

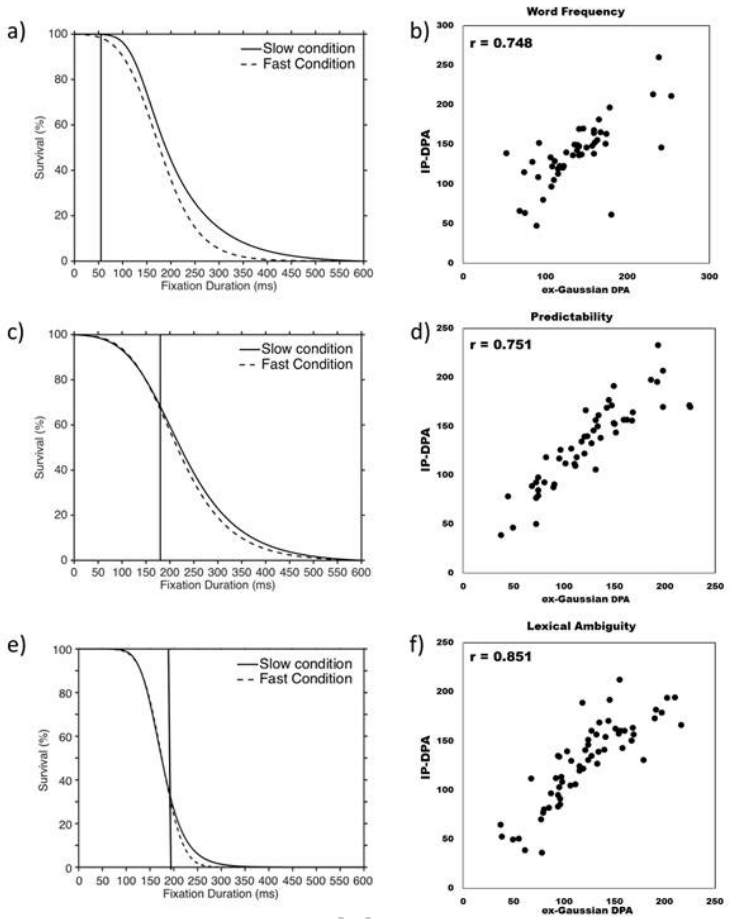
Table 1

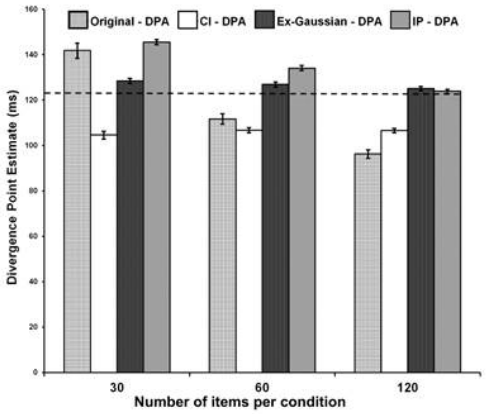
Variable	Study	fixation duration (ms)			DPA Procedure			
		Slow	Fast	Diff.	Original	CI-DPA	IP-DPA	ex-Gaussian
Word Frequency (Low – High Frequency)	Reingold et al. (2012) Valid Preview	234	214	20	145	138 (131–147)	139 (59/60)	136 (58/60)
Predictability (Low – High Predictability)	Sheridan & Reingold (2012a)	216	208	8	140	124 (101–142)	138 (55/60)	127 (49/60)
Lexical Ambiguity (Subordinate – Dominant Context)	Sheridan & Reingold (2012b)	228	216	12	139	121 (96–150)	131 (53/60)	122 (53/60)

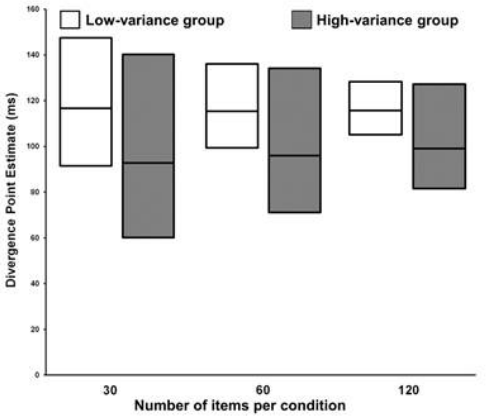
Summary of the re-analysis of data from prior reading studies by DPA procedure

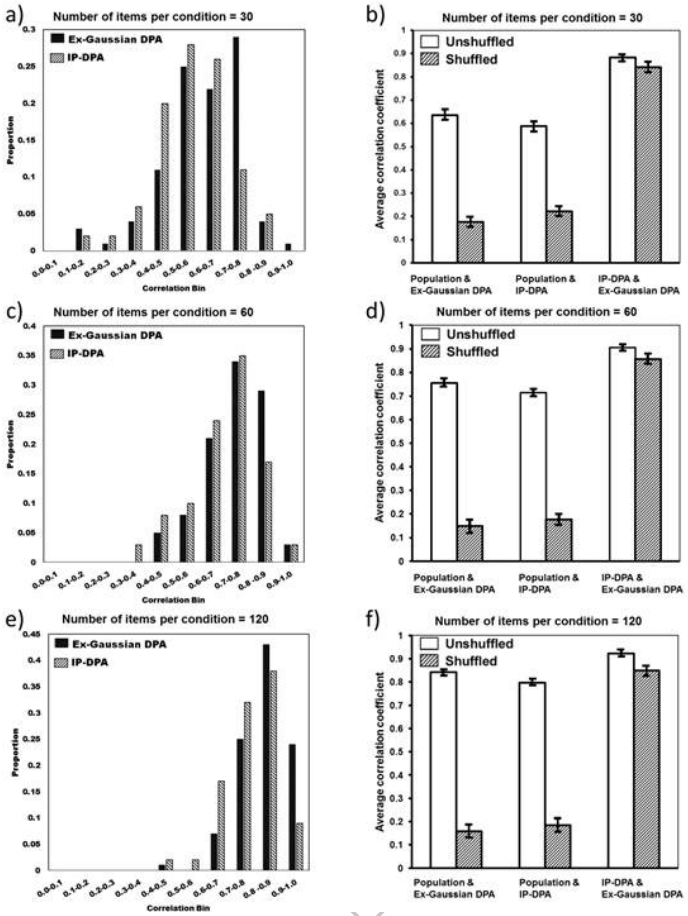
Note: DPA= Divergence Point Analysis; for fixation duration Diff. = slow minus fast fixation duration; CI-DPA = Confidence Interval DPA procedure; IP-DPA = Individual Participant DPA procedure; for the CI-DPA procedure, the confidence interval is shown in brackets below the estimate; for the IP-DPA and ex-Gaussian DPA procedures, the number of participants out of 60 for which a valid estimate was produced is shown in brackets below the estimate.

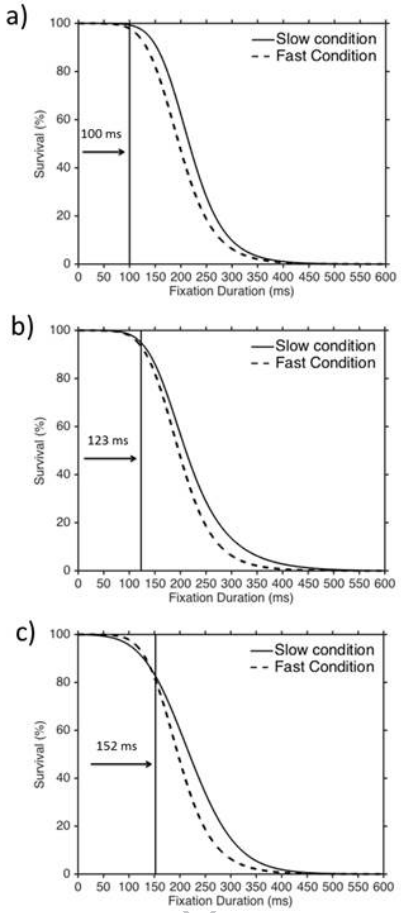

a)
Density
Fixation Duration (ms)
Gaussian
Exponential
ex-Gaussian
b)
100
0
Survival (%)
Fixation Duration (ms)
Gaussian
Exponential
ex-Gaussian
c)
Density
Fixation Duration (ms)
Gaussian
Exponential
ex-Gaussian
d)
100
0
Survival (%)
Fixation Duration (ms)
Gaussian
Exponential
ex-Gaussian
e)
Density
Fixation Duration (ms)
Gaussian
Exponential
ex-Gaussian
f)
100
0
Survival (%)
Fixation Duration (ms)
Gaussian
Exponential
ex-Gaussian


a)
Slow condition
Fast Condition
Survival (%)
Fixation Duration (ms)
b)
Word Frequency
r = 0.748
IP-DPA
ex-Gaussian DPA
c)
Slow condition
Fast Condition
Survival (%)
Fixation Duration (ms)
d)
Predictability
r = 0.751
IP-DPA
ex-Gaussian DPA
e)
Slow condition
Fast Condition
Survival (%)
Fixation Duration (ms)
f)
Lexical Ambiguity
r = 0.851
IP-DPA
ex-Gaussian DPA


Original - DPA
CI - DPA
Ex-Gaussian - DPA
IP - DPA
Divergence Point Estimate (ms)
0
20
40
60
80
100
120
140
160
30
60
120
Number of items per condition


Low-variance group
High-variance group
Divergence Point Estimate (ms)
0
20
40
60
80
100
120
140
160
30
60
120
Number of items per condition


a)
Number of items per condition = 30
Ex-Gaussian DPA
IP-DPA
Proportion
Correlation Bin
b)
Number of items per condition = 30
Unshuffled
Shuffled
Average correlation coefficient
Population & Ex-Gaussian DPA
Population & IP-DPA
IP-DPA & Ex-Gaussian DPA
c)
Number of items per condition = 60
Ex-Gaussian DPA
IP-DPA
Proportion
Correlation Bin
d)
Number of items per condition = 60
Unshuffled
Shuffled
Average correlation coefficient
Population & Ex-Gaussian DPA
Population & IP-DPA
IP-DPA & Ex-Gaussian DPA
e)
Number of items per condition = 120
Ex-Gaussian DPA
IP-DPA
Proportion
Correlation Bin
f)
Number of items per condition = 120
Unshuffled
Shuffled
Average correlation coefficient
Population & Ex-Gaussian DPA
Population & IP-DPA
IP-DPA & Ex-Gaussian DPA


a)
Slow condition
Fast Condition
100 ms
Survival (%)
Fixation Duration (ms)
b)
Slow condition
Fast Condition
123 ms
Survival (%)
Fixation Duration (ms)
c)
Slow condition
Fast Condition
152 ms
Survival (%)
Fixation Duration (ms)